Mathematical Analysis for Engineers

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Foreword

This book is a translation of the third French edition. It is intended for engineering students who followed a basic course in analysis (differential and integral calculus). It corresponds to a second year course at the Ecole Polytechnique Fédérale of Lausanne. It can also be useful, as a complement to a more theoretical course, to mathematics and physics students.

There are excellent books on the matters that are discussed here; some of them that we particularly like are mentioned in the bibliography. Our approach is, however, different. We have emphasized the learning of the field through examples and exercises. The theoretical part is short, definitions and theorems are given without comments.

The book is organized as follows. The first three parts (Vector analysis, Complex analysis and Fourier analysis) represent the theoretical part and they are essentially independent of each other. The fourth part gives detailed solutions to all exercises that are proposed in the first three parts. The theoretical discussion follows the following pattern.

1) Definitions and theorems are stated, with mathematical rigor, but without comments. We also mention the precise pages of certain books from the bibliography where the interested reader can find further developments.

2) Some significant examples are discussed in detail.

3) Finally, several exercises are given and, as already said, solved in the fourth part of the book. The first type of exercise will help students to master the concepts and the techniques. A second type (identified with a *) presents some theoretical developments that allow the more motivated students to deepen their understanding of the subject.

We would now like to make some comments on the bibliography. We have selected two types of book.

1) As mathematical references, we particularly like the following books:
- for vector analysis the Protter–Morrey book and the more advanced Fleming book;
- for complex analysis the very classical Ahlfors book;
- for Fourier series the already mentioned Protter–Morrey book, while for Fourier and Laplace transforms the Widder book;
- the two Stein–Shakarchi books cover a large part of the matters discussed here (complex and Fourier analysis);
- finally, in French, the three volumes of Chatterji cover in detail the entire subject of our book.

2) For engineers we recommend the Kreyszig book. The two small books, in French, by Arbenz–Wohlhauser are also nice as a short introduction.

We have benefited from several comments from students and colleagues; notably S. Bandyopadhyay, M. Cibils, G. Croce, G. Csato, J. Douchet, H. Gebran, O. Kneuss, P. Metzener, G. Pisante, A. Ribeiro, L. Rollaz and K. D. Semmler. The translation and the preparation of the English version have been carried out by R. Guglielmetti.