

# Lecture Notes in Mathematics

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Bernard Dacorogna

Weak Continuity  
and Weak Lower Semicontinuity  
of Non-Linear Functionals

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**Author**

Bernard Dacorogna  
Département de Mathématiques  
Ecole Polytechnique Fédérale de Lausanne  
61, Avenue de Cour, 1007 Lausanne, Switzerland

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## PREFACE

These notes are the result of a graduate course given at Brown during the first quarter of 1981. They should be considered as an introduction to the subject. They are not intended to be a complete presentation of all the results in this area. The results presented here are not all new and obviously a large part of the first and second chapter owes much to various works of F. Murat and L. Tartar on compensated compactness.

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B. Dacorogna  
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WEAK CONTINUITY AND WEAK LOWER SEMICONTINUITY OF NON-LINEAR FUNCTIONALS

by

B. Dacorogna

ABSTRACT

These notes deal with the behavior of nonlinear functionals with respect to weak convergence. In the first chapter we investigate several necessary and sufficient conditions in order that a nonlinear function is weakly continuous or weakly lower semicontinuous. In Chapter II we give some applications of the results of Chapter I to partial differential equations and to nonlinear elasticity. In the last chapter we deal with dual and relaxed variational problems.

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