

Mathematical Analysis for Engineers

B. Dacorogna and C. Tanteri

Contents

Foreword	vi
I Vector analysis	1
1 Differential operators of mathematical physics	3
1.1 Definitions and theoretical results	3
1.2 Examples	5
1.3 Exercises	7
2 Line integrals	9
2.1 Definitions and theoretical results	9
2.2 Examples	10
2.3 Exercises	11
3 Gradient vector fields	13
3.1 Definitions and theoretical results	13
3.2 Examples	14
3.3 Exercises	18
4 Green theorem	21
4.1 Definitions and theoretical results	21
4.2 Examples	22
4.3 Exercises	24
5 Surface integrals	27
5.1 Definitions and theoretical results	27
5.2 Examples	29
5.3 Exercises	31
6 Divergence theorem	33
6.1 Definitions and theoretical results	33
6.2 Examples	34

6.3 Exercises	36
7 Stokes theorem	39
7.1 Definitions and theoretical results	39
7.2 Examples	41
7.3 Exercises	43
8 Appendix	45
8.1 Some notations and notions of topology	45
8.2 Some notations for functional spaces	49
8.3 Curves	50
8.4 Surfaces	52
8.5 Change of variables	64
II Complex analysis	67
9 Holomorphic functions and Cauchy–Riemann equations	69
9.1 Definitions and theoretical results	69
9.2 Examples	71
9.3 Exercises	74
10 Complex integration	77
10.1 Definitions and theoretical results	77
10.2 Examples	78
10.3 Exercises	79
11 Laurent series	83
11.1 Definitions and theoretical results	83
11.2 Examples	86
11.3 Exercises	88
12 Residue theorem and applications	91
12.1 Part I	91
12.1.1 Definitions and theoretical results	91
12.1.2 Examples	92
12.2 Part II: Evaluation of real integrals	93
12.3 Exercises	97
13 Conformal mapping	101
13.1 Definitions and theoretical results	101
13.2 Examples	102
13.3 Exercises	104

III	Fourier analysis	107
14	Fourier series	109
14.1	Definitions and theoretical results	109
14.2	Examples	113
14.3	Exercises	116
15	Fourier transform	121
15.1	Definitions and theoretical results	121
15.2	Examples	123
15.3	Exercises	125
16	Laplace transform	127
16.1	Definitions and theoretical results	127
16.2	Examples	129
16.3	Exercises	132
17	Applications to ordinary differential equations	135
17.1	Cauchy problem	135
17.2	Sturm–Liouville problem	137
17.3	Some other examples solved by Fourier analysis	140
17.4	Exercises	143
18	Applications to partial differential equations	145
18.1	Heat equation	145
18.2	Wave equation	150
18.3	Laplace equation in a rectangle	152
18.4	Laplace equation in a disk	155
18.5	Laplace equation in a simply connected domain	159
18.6	Exercises	162
IV	Solutions to the exercises	167
1	Differential operators of mathematical physics	169
2	Line integrals	177
3	Gradient vector fields	181
4	Green theorem	189
5	Surface integrals	199

6	Divergence theorem	203
7	Stokes theorem	219
9	Holomorphic functions and Cauchy–Riemann equations	233
10	Complex integration	239
11	Laurent series	247
12	Residue theorem and applications	263
13	Conformal mapping	277
14	Fourier series	291
15	Fourier transform	303
16	Laplace transform	309
17	Applications to ordinary differential equations	317
18	Applications to partial differential equations	331
	Bibliography	353
	Table of Fourier Transform	355
	Table of Laplace Transform	356
	Index	357

Foreword

This book is a translation of the third French edition. It is intended for engineering students who followed a basic course in analysis (differential and integral calculus). It corresponds to a second year course at the Ecole Polytechnique Fédérale of Lausanne. It can also be useful, as a complement to a more theoretical course, to mathematics and physics students.

There are excellent books on the matters that are discussed here; some of them that we particularly like are mentioned in the bibliography. Our approach is, however, different. We have emphasized the learning of the field through examples and exercises. The theoretical part is short, definitions and theorems are given without comments.

The book is organized as follows. The first three parts (Vector analysis, Complex analysis and Fourier analysis) represent the theoretical part and they are essentially independent of each other. The fourth part gives detailed solutions to all exercises that are proposed in the first three parts. The theoretical discussion follows the following pattern.

- 1) Definitions and theorems are stated, with mathematical rigor, but without comments. We also mention the precise pages of certain books from the bibliography where the interested reader can find further developments.

- 2) Some significant examples are discussed in detail.

- 3) Finally, several exercises are given and, as already said, solved in the fourth part of the book. The first type of exercise will help students to master the concepts and the techniques. A second type (identified with a *) presents some theoretical developments that allow the more motivated students to deepen their understanding of the subject.

We would now like to make some comments on the bibliography. We have selected two types of book.

- 1) As mathematical references, we particularly like the following books:

- for vector analysis the Protter–Morrey book and the more advanced Fleming book;
- for complex analysis the very classical Ahlfors book;
- for Fourier series the already mentioned Protter–Morrey book, while for Fourier and Laplace transforms the Widder book;
- the two Stein–Shakarchi books cover a large part of the matters discussed here (complex and Fourier analysis);
- finally, in French, the three volumes of Chatterji cover in detail the entire subject of our book.

2) For engineers we recommend the Kreyszig book. The two small books, in French, by Arbenz–Wohlhauser are also nice as a short introduction.

We have benefited from several comments from students and colleagues; notably S. Bandyopadhyay, M. Cibils, G. Croce, G. Csato, J. Douchet, H. Gebran, O. Kneuss, P. Metzener, G. Pisante, A. Ribeiro, L. Rollaz and K. D. Semmler. The translation and the preparation of the English version have been carried out by R. Guglielmetti.